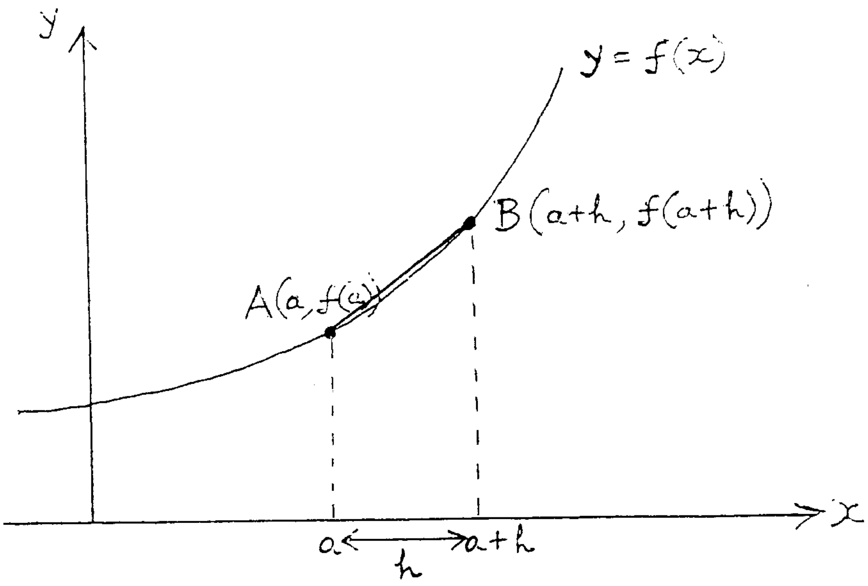
BASIC DIFFERENTIATION

DIFFERENTIATION FROM FIRST PRINCIPLES

Recall that $f'(a)$ is the gradient of the tangent to the curve $y = f(x)$ at the point where $x = a$.



$$m_{AB} = \frac{f(a+h) - f(a)}{a+h-a} = \frac{f(a+h) - f(a)}{h}$$

If h is small, the gradient of the chord AB will be approximately equal to the gradient of the tangent at A. As h gets smaller, the approximation becomes more accurate.

As $h \to 0$, $\dfrac{f(a+h) - f(a)}{h} \to f'(a)$.

We write $f'(a) = \lim_{h \to 0} \left\{ \dfrac{f(a+h) - f(a)}{h} \right\}$.

Replacing a with x gives a formula for finding $f'(x)$:

$$f'(x) = \lim_{h \to 0} \left\{ \frac{f(x+h) - f(x)}{h} \right\}$$

Using the above formula to find $f'(x)$ is known as **differentiation from first principles**.

Worked Example 1

Find the derivative of the function $f(x) = 3x^2$ **from first principles.**

Solution

$$f(x) = 3x^2$$

$$f(x+h) = 3(x+h)^2 = 3(x^2 + 2xh + h^2)$$
$$= 3x^2 + 6xh + 3h^2$$

$$\frac{f(x+h) - f(x)}{h} = \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$$
$$= \frac{6xh + 3h^2}{h}$$
$$= 6x + 3h$$

$$f'(x) = \lim_{h \to 0} \left\{ \frac{f(x+h) - f(x)}{h} \right\}$$
$$= \lim_{h \to 0} \{6x + 3h\} = 6x$$

Worked Example 2

Find the derivative of the function $f(x) = 2x^2 - 3x + 1$ **from first principles**.

Solution

$$f(x) = 2x^2 - 3x + 1$$

$$\begin{aligned} f(x+h) &= 2(x+h)^2 - 3(x+h) + 1 \\ &= 2(x^2 + 2xh + h^2) - 3(x+h) + 1 \\ &= 2x^2 + 4xh + 2h^2 - 3x - 3h + 1 \end{aligned}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - (2x^2 - 3x + 1)}{h} \\ &= \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 1 - 2x^2 + 3x - 1}{h} \\ &= \frac{4xh + 2h^2 - 3h}{h} \\ &= 4x + 2h - 3 \end{aligned}$$

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \left\{ \frac{f(x+h) - f(x)}{h} \right\} \\ &= \lim_{h \to 0} \{4x + 2h - 3\} = 4x - 3 \end{aligned}$$